CS 6501 Natural Language Processing

Recurrent Neural Networks Language Models

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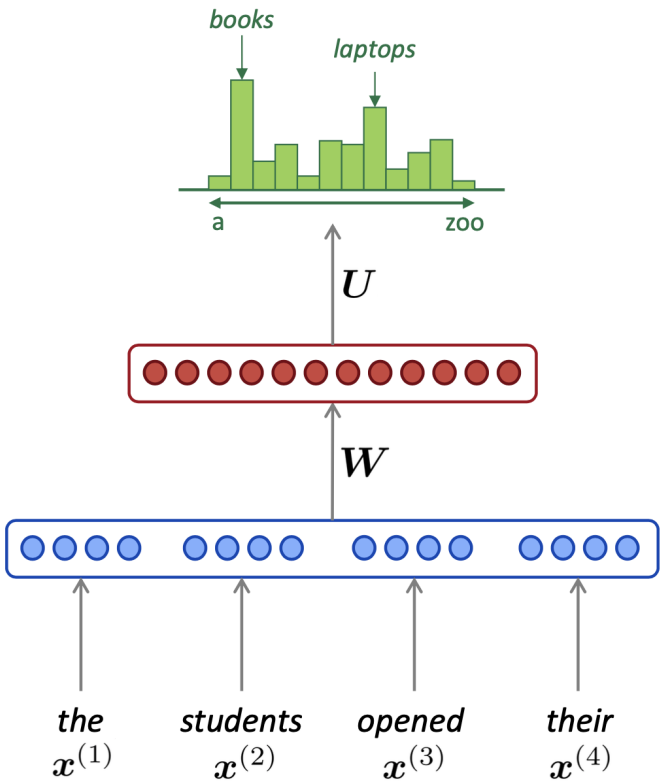
Overview

1. Neural Network Language Models
2. Recurrent Neural Networks
3. RNN Language Modeling
4. Challenge of Training RNNs

Neural Network Language Models

A Neural Language Model with Fixed Window Size



- Word indices: x_1, x_2, x_3, x_4

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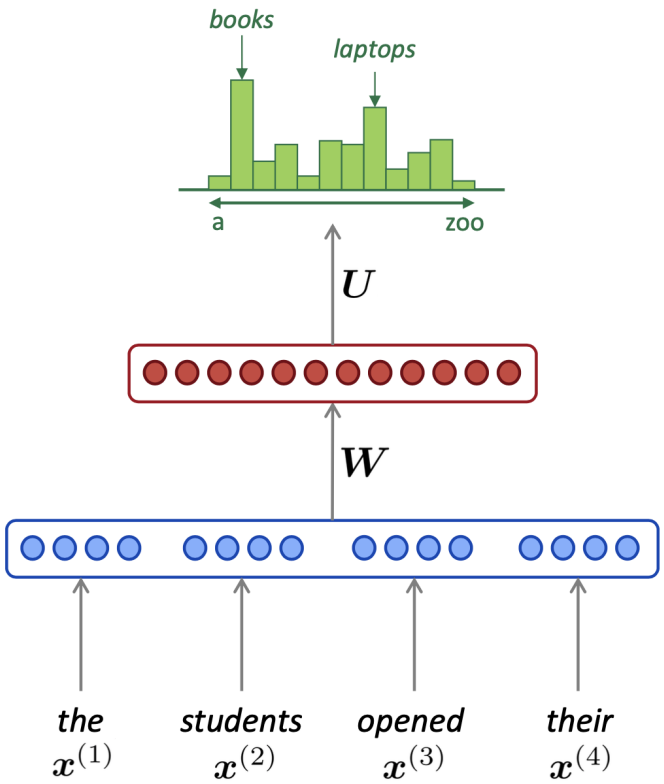

- Concatenated word embeddings

$$v^\top = [v_{x_1}^\top, v_{x_2}^\top, v_{x_3}^\top, v_{x_4}^\top] \qquad (3)$$

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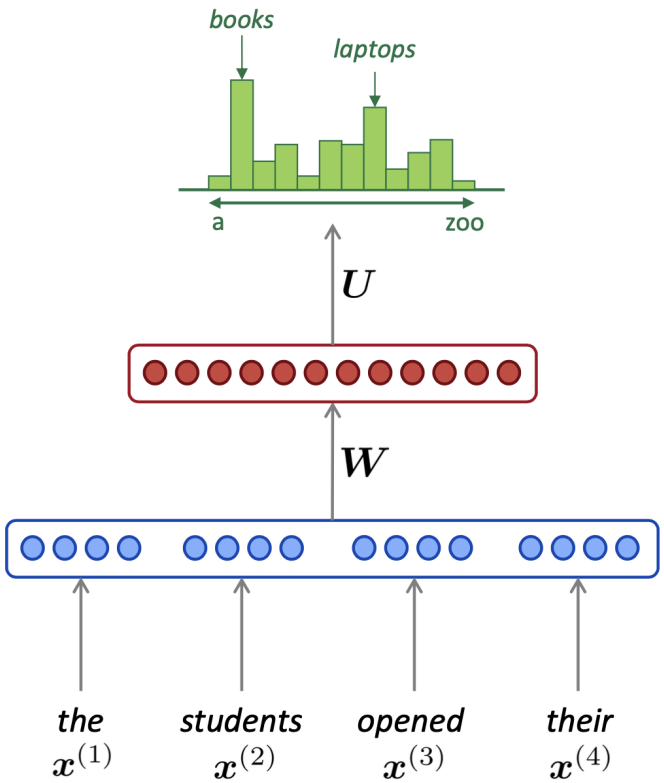

- Hidden layer: $f(\cdot)$ could be any nonlinear activation function

$$h = f(Wv + b_1) \tag{2}$$

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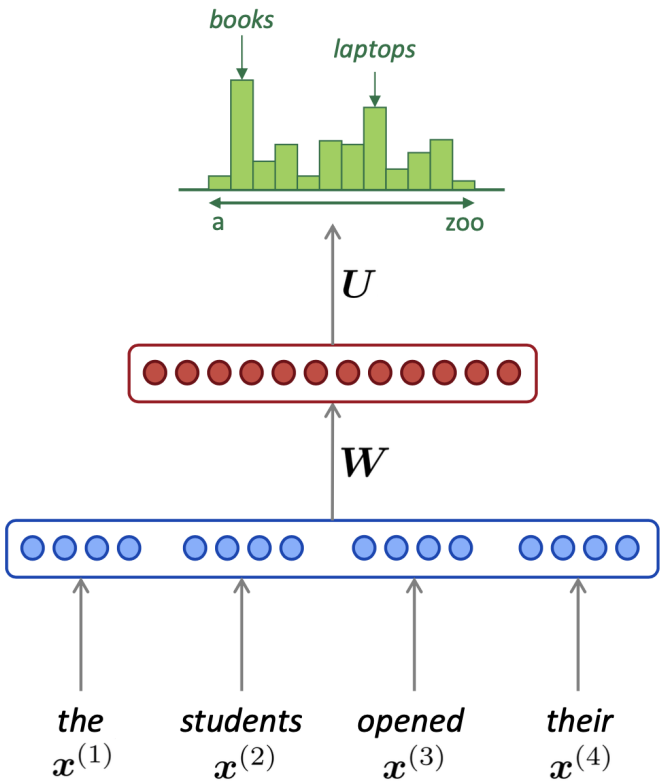

- Output distribution

$$P(X_5 \mid \boldsymbol{X}_{1:4}) = \text{softmax}(\boldsymbol{U}\boldsymbol{h}+\boldsymbol{b}_2) \in \mathbb{R}^{|\mathcal{V}|} \quad (1)$$

- Hidden layer: $f(\cdot)$ could be any nonlinear activation function

$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{v} + \boldsymbol{b}_1) \quad (2)$$

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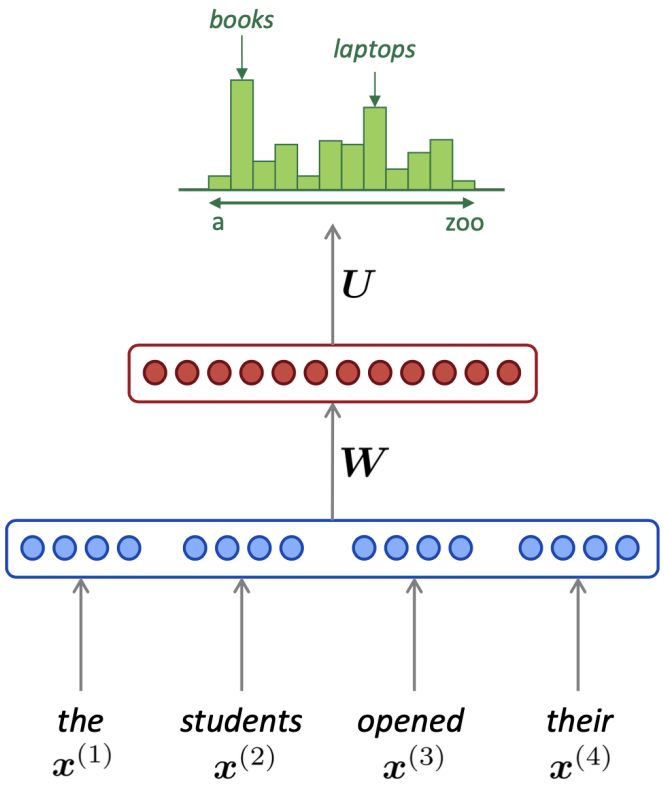

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- Word indices: x_1, x_2, x_3, x_4

This is the very first neural neural language model [Bengio et al., 2001], which has a similar network architecture as the one discussed in lecture 03.

A Neural Probabilistic Language Model

The first paragraph of the paper *A Neural Probabilistic Language Model* [Bengio et al., 2001]

1 Introduction

A fundamental problem that makes language modeling and other learning problems difficult is the *curse of dimensionality*. It is particularly obvious in the case when one wants to model the joint distribution between many discrete random variables (such as words in a sentence, or discrete attributes in a data-mining task). For example, if one wants to model the joint distribution of 10 consecutive words in a natural language with a vocabulary V of size 100,000, there are potentially $100\,000^{10} - 1 = 10^{50} - 1$ free parameters.

[1]For more precise description, please refer to [Shalev-Shwartz and Ben-David, 2014]

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Curse of dimensionality

The sample complexity is an exponential function of the dimensionality of data[1]

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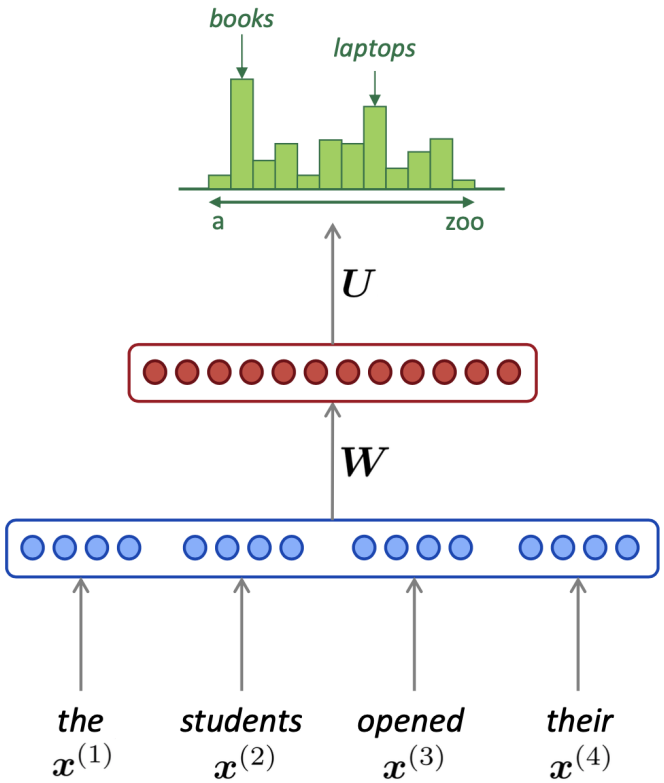

Improvement over n-gram language models

- Less parameters (with large n's)
- No sparsity problem
- No smoothing is needed

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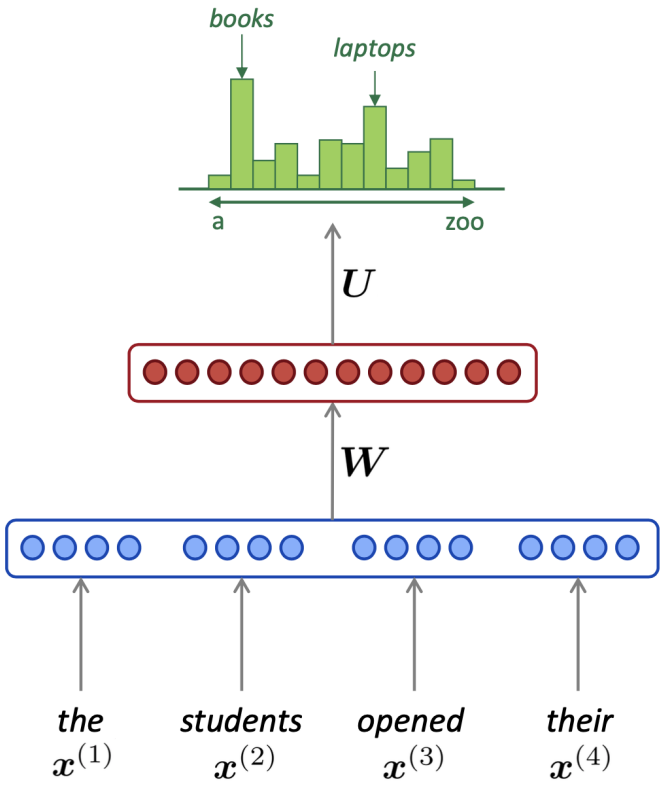

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Remaining issues

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- Same word will be computed k times along the sliding window, where k is the window size

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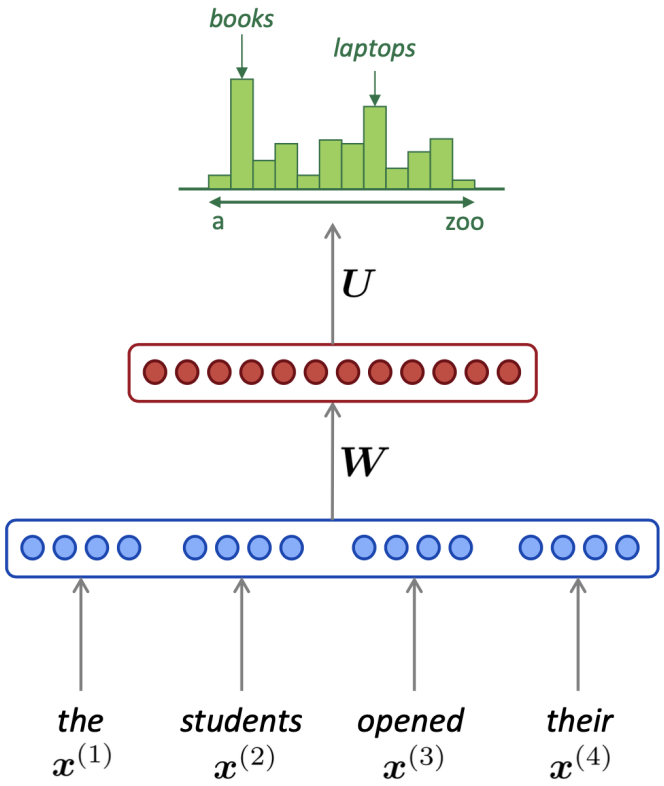

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We need a new neural network architecture that can read words continuously along predictions

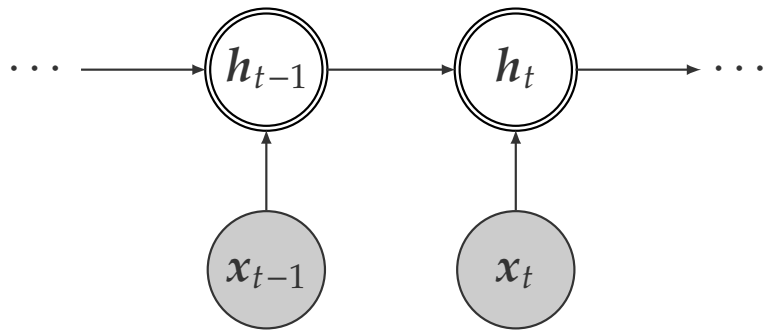
Recurrent Neural Networks

Recurrent Neural Networks (RNNs)

A simple RNN is defined by the following recursive function

$$h_t = f(x_t, h_{t-1}) \tag{4}$$

and depicted as



where

- h_{t-1}: hidden state at time step $t-1$
- x_t: input at time step t
- h_t: hidden state at time step t

A Simple Transition Function

In the simplest case, the transition function f is defined with an element-wise Sigmoid function and a linear transformation of x_t and h_{t-1}

$$h_t = f(x_t, h_{t-1}) = \sigma(\mathbf{W}_h h_{t-1} + \mathbf{W}_i x_t + b) \tag{5}$$

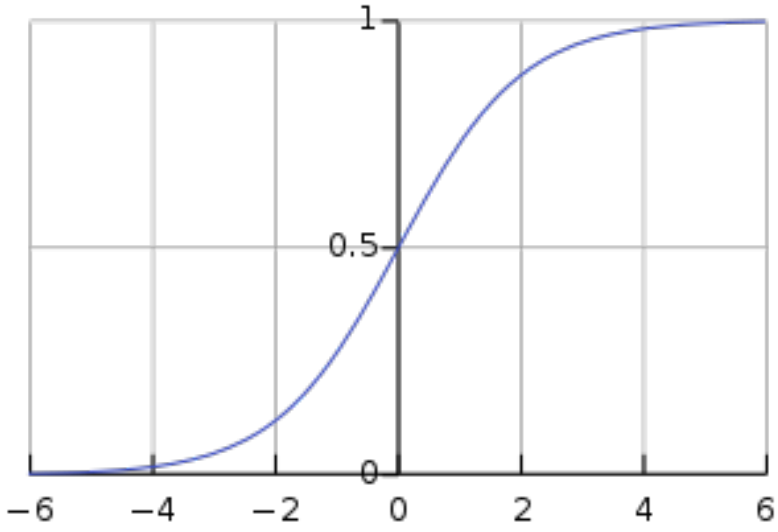
where

- x_t: input word embedding
- h_{t-1}: hidden statement from previous time step
- $\mathbf{W}_h$: parameter matrix for hidden states
- $\mathbf{W}_i$: parameter matrix for inputs
- b: bias term (also a parameter)

Sigmoid Function

A Sigmoid function with one-dimensional input $x \in (-\infty, \infty)$

$$\sigma(x) = \frac{1}{1 - e^{-x}}$$



The potential numeric issue caused by the Sigmoid function

- $\sigma(x) \rightarrow 1$ with $x \gg 6$
- $\sigma(x) \rightarrow 0, x \ll -6$

The output of the Sigmoid function will approximate a constant, when the input value is beyond certain ranges

Unfolding RNNs

We can unfold this recursive definition of a RNN

$$\boldsymbol{h}_t = \boldsymbol{f}(\boldsymbol{x}_t, \boldsymbol{h}_{t-1}) \tag{6}$$

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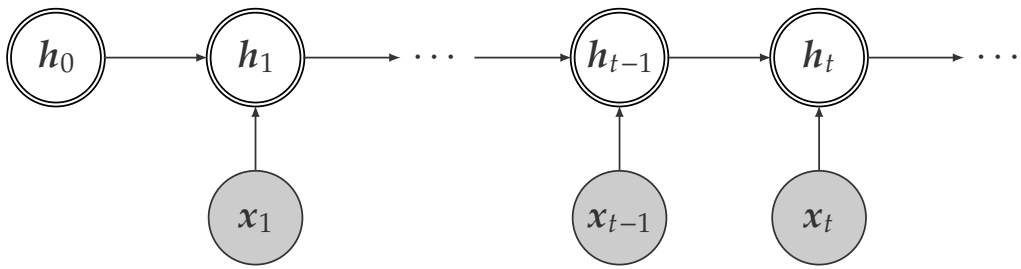
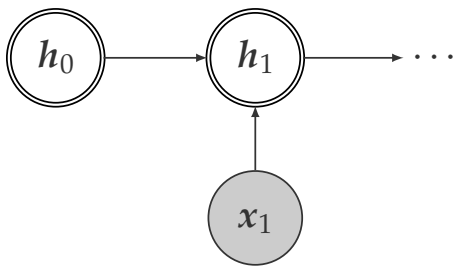


$$\begin{aligned}
\boldsymbol{h}_t &= f(\boldsymbol{x}_t, f(\boldsymbol{x}_{t-1}, \boldsymbol{h}_{t-2})) \\
&= f(\boldsymbol{x}_t, f(\boldsymbol{x}_{t-1}, f(\boldsymbol{x}_{t-2}, \boldsymbol{h}_{t-3}))) \\
&= \cdots \\
&= f(\boldsymbol{x}_t, f(\boldsymbol{x}_{t-1}, f(\boldsymbol{x}_{t-2}, \cdots f(\boldsymbol{x}_1, \boldsymbol{h}_0) \cdots))) \qquad (7)
\end{aligned}$$

Base Condition

Base condition defines the starting point of the recursive computation

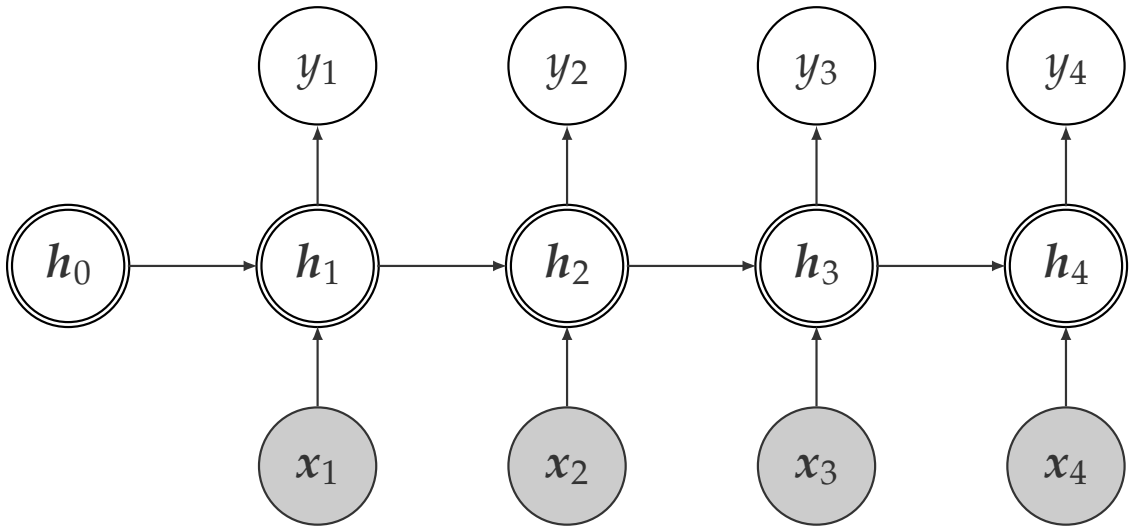


$$h_t = f(x_t, f(x_{t-1}, f(x_{t-2}, \cdots f(\boldsymbol{x_1}, \boldsymbol{h_0}) \cdots))) \tag{8}$$

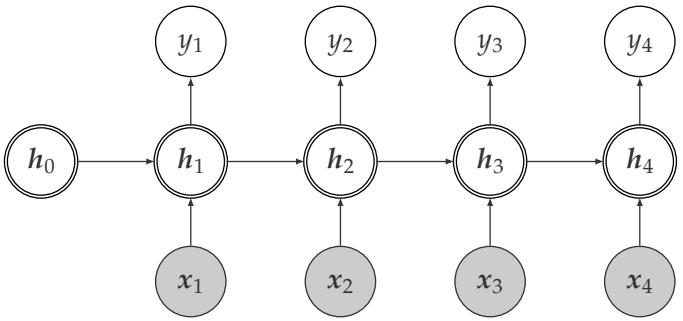
- h_0: zero vector or parameter
- x_1: input at time $t = 1$

RNN for Sequential Prediction

In general, RNNs can be used for any sequential modeling tasks



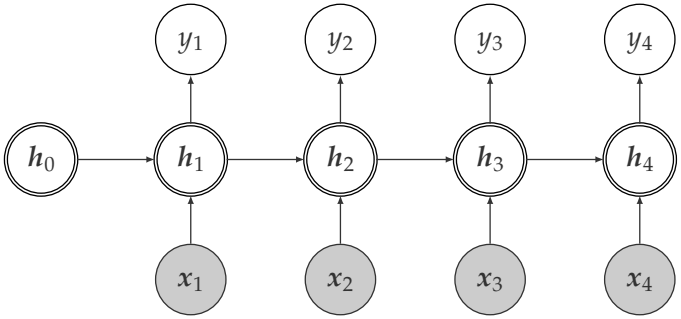
Sequential Modeling as Classification



- Prediction at each time step t

$$\hat{y}_t = \underset{y}{\operatorname{argmax}}\, P(y; \boldsymbol{h}_t) \tag{9}$$

Sequential Modeling as Classification



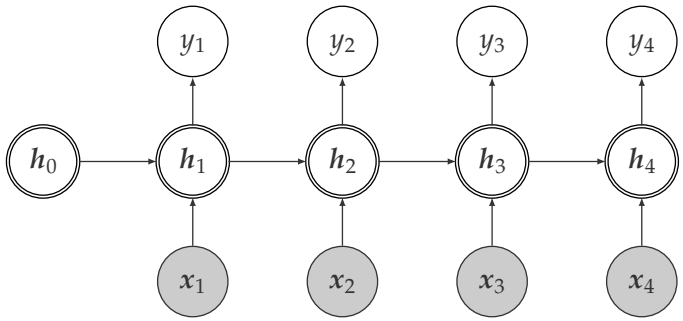
- Prediction at each time step t

$$\hat{y}_t = \underset{y}{\operatorname{argmax}}\, P(y; \boldsymbol{h}_t) \tag{9}$$

- Loss at single time step t

$$L_t(y_t, \hat{y}_t) = -\log P(y_t; \boldsymbol{h}_t) \tag{10}$$

Sequential Modeling as Classification



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- Loss at single time step t

$$L_t(y_t, \hat{y}_t) = -\log P(y_t; \boldsymbol{h}_t) \tag{10}$$

- The total loss

$$\ell = \sum_{t=1}^{T} L_t(y_t, \hat{y}_t) \tag{11}$$

RNN Language Modeling

Language Models

A language model defines the probability of x_t given $\boldsymbol{x} = (x_1, x_2, \ldots, x_{t-1})$ as

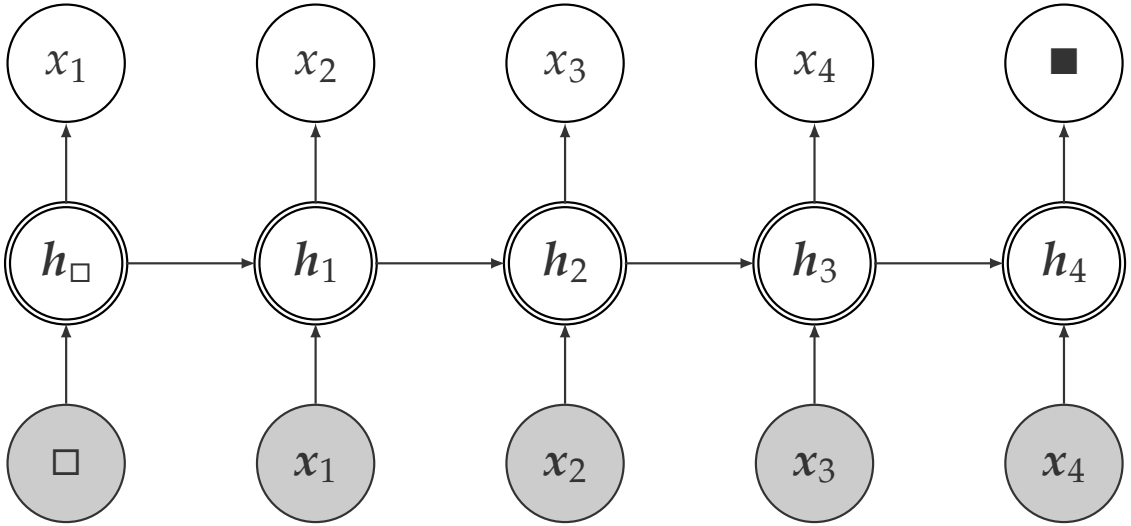
$$P(x_t \mid x_1, \ldots, x_{t-1}) \tag{12}$$

and the joint probability as

$$\begin{aligned} P(\boldsymbol{x}_{1:T}) \quad = \quad & P(x_1) \cdot P(x_2 \mid x_1) \\ & \cdot \ \ldots \ \cdot \\ & \cdot P(x_T \mid x_1, x_2, \ldots, x_{T-1}) \end{aligned}$$

Language Modeling with RNNs

Using RNNs for language modeling

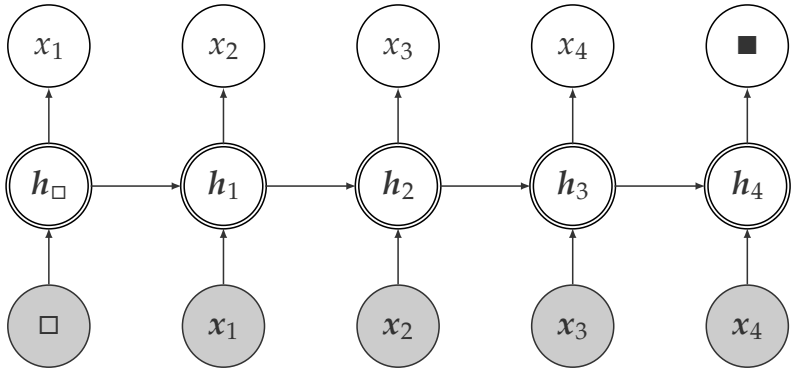


with two special tokens

$$\{\square, x_1, \ldots, x_T, \blacksquare\}$$

RNN Language Models

For a given sentence $\{x_1, \ldots, x_t\}$, the input at time t is word embedding $\boldsymbol{x}_t$



The probability distribution of next word X_t

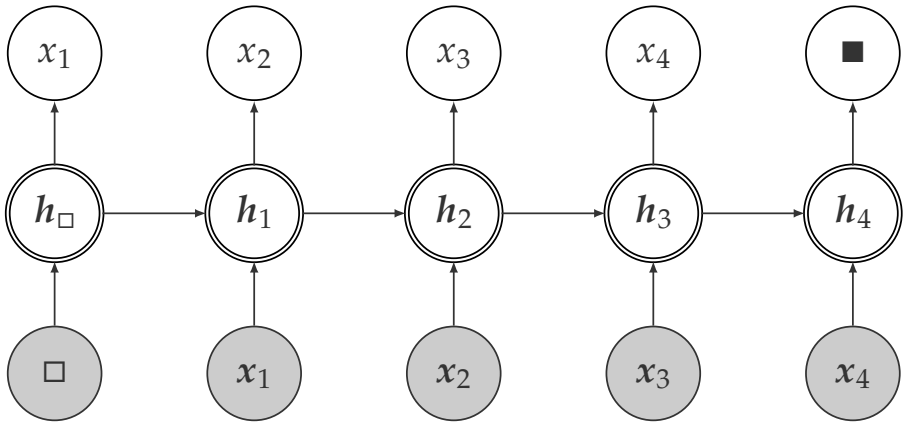
$$P(X_t = x \mid \boldsymbol{x}_{1:t-1}) = \frac{\exp(\boldsymbol{w}_{o,x}^\top \boldsymbol{h}_{t-1})}{\sum_{x' \in \mathcal{V}} \exp(\boldsymbol{w}_{o,x'}^\top \boldsymbol{h}_{t-1})} \tag{13}$$

where

- $\boldsymbol{w}_{o,x}$ is the output weight vector (parameter) associated with word x
- $\mathcal{V}$ is the word vocabulary

Special Cases

Similar to statistical language modeling, there are also two special cases that we need to consider



$$\{□, x_1, \ldots, x_T, ■\}$$

The corresponding prediction functions are defined as

- At time $t = 1$

$$P(X_1 = x) \propto \exp(\boldsymbol{w}_{o,x}^\top \boldsymbol{h}_□) \tag{14}$$

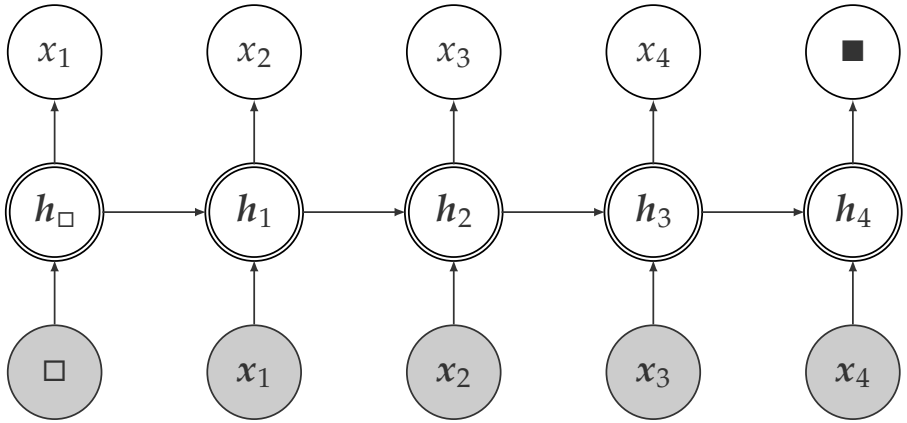
- At time $t = T$

$$P(X_T = ■ \mid \boldsymbol{x}_{1:T-1}) \propto \exp(\boldsymbol{w}_{o,x}^\top \boldsymbol{h}_{T-1}) \tag{15}$$

Challenge of Training RNNs

Objective

The training objective for each timestep is to predict the next token in the text

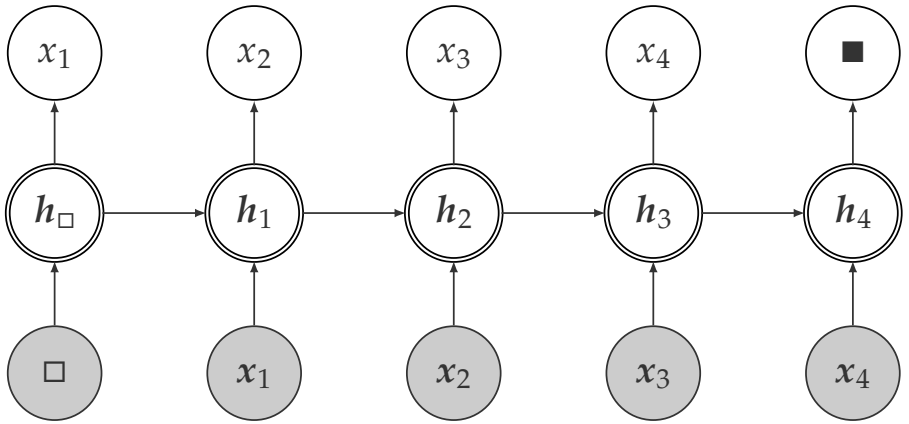


- Prediction at step t, $P(X_t = x \mid \boldsymbol{x}_{1:t-1}) = \frac{\exp(\boldsymbol{w}_{o,x}^\mathsf{T} \boldsymbol{h}_{t-1})}{\sum_{x' \in \mathcal{V}} \exp(\boldsymbol{w}_{o,x'}^\mathsf{T} \boldsymbol{h}_{t-1})}$
- Loss at step t, $L_t = -\log P(X_t = x \mid \boldsymbol{x}_{1:t-1})$

Gradients

Let $\boldsymbol{\theta}$ denote all model parameters

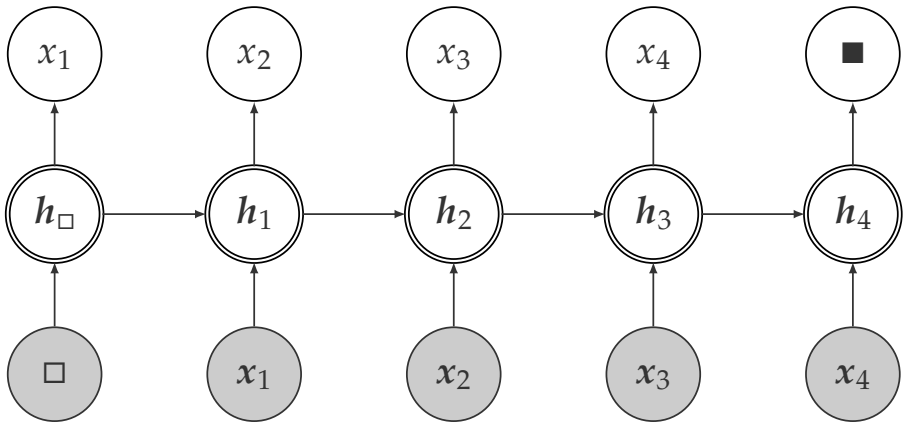
$$\frac{\partial \ell}{\partial \boldsymbol{\theta}} = \sum_{t=1}^{T} \frac{\partial L_t}{\partial \boldsymbol{\theta}} \tag{16}$$



Backpropagation Through Time [Rumelhart et al., 1985, BPTT]

Model Parameters

Before computing the gradient of each L_t with respect to model parameters, let us count how many parameters that we need consider



- Output parameter matrix $\boldsymbol{W}_o = (\boldsymbol{w}_{o,1}, \ldots, \boldsymbol{w}_{o,V})$

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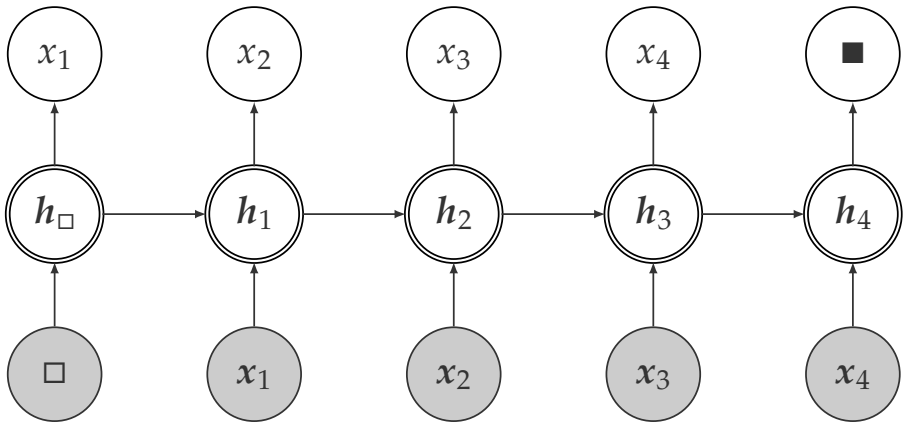

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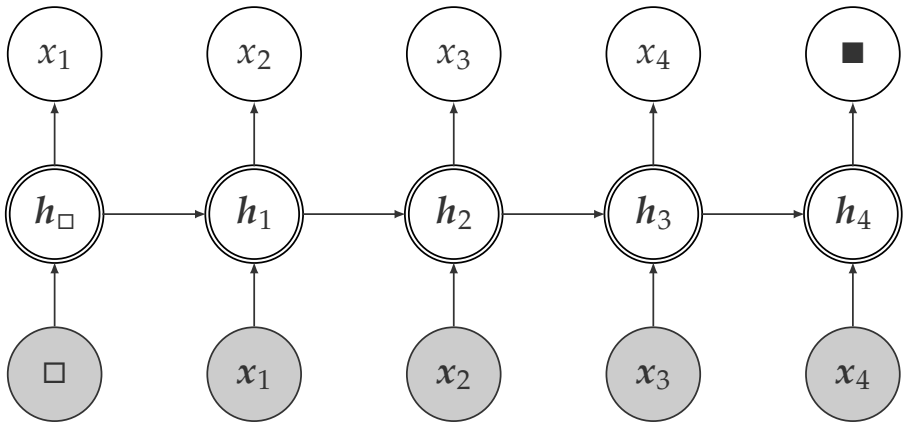

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- Input word embedding matrix $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_V)$
- Neural network parameters $\boldsymbol{W}_h, \boldsymbol{W}_i, \boldsymbol{b}$

Backpropagation Through Time

Take time step t as an example, we can take a look the gradient computation of some specific parameters

- Output model parameter $\frac{\partial L_t}{\partial w_{o,\cdot}}$

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- Neural network parameters, for example $\boldsymbol{W}_h$

$$\frac{\partial L_t}{\partial \boldsymbol{W}_h} = \sum_{i=1}^{t} \{ \frac{\partial L_t}{\partial \boldsymbol{h}_t} \cdot (\prod_{j=i}^{t-1} \frac{\partial \boldsymbol{h}_{j+1}}{\partial \boldsymbol{h}_j}) \cdot \frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{W}_h} \} \tag{17}$$

Similar patterns for the other two neural network parameters $\boldsymbol{W}_i$ and $\boldsymbol{b}$

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  Similar patterns for the other two neural network parameters $\boldsymbol{W}_i$ and $\boldsymbol{b}$

- Word embedding $\frac{\partial L_t}{\partial \boldsymbol{x}_{t'}}$
  - E.g., word embedding $\boldsymbol{x}_{t'}$ is the input of $\boldsymbol{h}_t$ if $t' \leq t$, so ...

Challenges

For each timestep, we need to compute the gradient using the chain rule:

$$\frac{\partial L_t}{\partial \boldsymbol{W}_h} = \sum_{i=1}^{t} \{ \frac{\partial L_t}{\partial \boldsymbol{h}_t} \cdot (\prod_{j=i}^{t-1} \frac{\partial \boldsymbol{h}_{j+1}}{\partial \boldsymbol{h}_j}) \cdot \frac{\partial \boldsymbol{h}_i}{\partial \boldsymbol{W}_h} \} \tag{18}$$

The chain rule of gradient will cause two potential problems in training RNNs

- vanishing gradients: $\frac{\partial L_t}{\partial \boldsymbol{\theta}} \to 0$
- exploding gradients: $\frac{\partial L_t}{\partial \boldsymbol{\theta}} \geq M$

[Pascanu et al., 2013]

Exploding Gradients

Solution: **norm clipping** [Pascanu et al., 2013].

Consider the gradient $g = \frac{\partial \ell}{\partial \theta}$,

$$\hat{g} \leftarrow \tau \cdot \frac{g}{\|g\|} \tag{19}$$

when $\|g\| > \tau$.

- Usually, $\tau = 3$ or 5 in practice.
- Smaller gradient will cause slower learning progress

Vanishing Gradients

Solution:

- initialize parameters carefully
- replace hidden state transition function $\sigma(\cdot)$ with other options

$$f(\boldsymbol{x}_t, \boldsymbol{h}_{t-1}) = \sigma(\mathbf{W}_h \boldsymbol{h}_{t-1} + \mathbf{W}_i \boldsymbol{x}_t + \boldsymbol{b}) \tag{20}$$

  - LSTM [Hochreiter and Schmidhuber, 1997]
  - GRU [Cho et al., 2014]

Long Short-Term Memory

From the first page of the original paper proposing LSTM [Hochreiter and Schmidhuber, 1997]

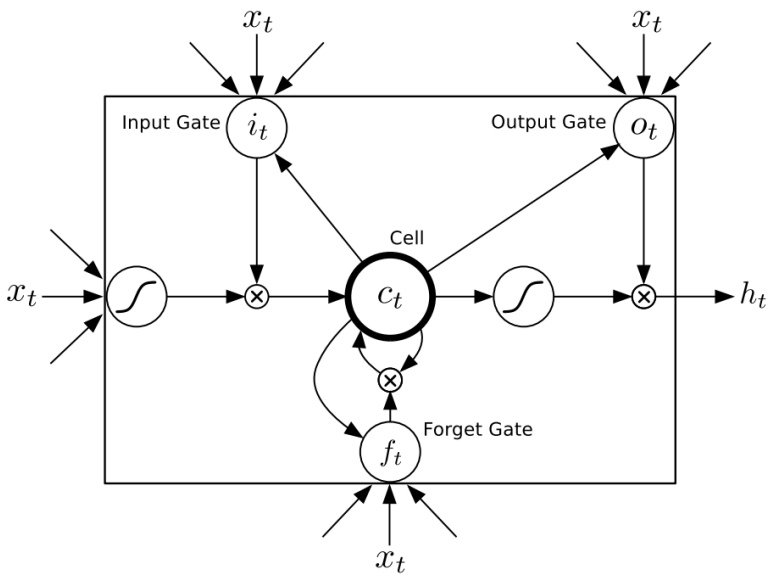
The problem. With conventional "Back-Propagation Through Time" (BPTT, e.g., Williams and Zipser 1992, Werbos 1988) or "Real-Time Recurrent Learning" (RTRL, e.g., Robinson and Fallside 1987), error signals "flowing backwards in time" tend to either (1) blow up or (2) vanish: the temporal evolution of the backpropagated error exponentially depends on the size of the weights (Hochreiter 1991). Case (1) may lead to oscillating weights, while in case (2) learning to bridge long time lags takes a prohibitive amount of time, or does not work at all (see section 3).

The remedy. This paper presents *"Long Short-Term Memory"* (LSTM), a novel recurrent network architecture in conjunction with an appropriate gradient-based learning algorithm. LSTM is designed to overcome these error back-flow problems. It can learn to bridge time intervals in excess of 1000 steps even in case of noisy, incompressible input sequences, without loss of short time lag capabilities. This is achieved by an efficient, gradient-based algorithm for an architecture

Long Short-Term Memory

Rather than directly taking input and hidden state as simple transition function, LSTM relies on three cates to control *how much* information it should take from input and hidden state before combining them together

$$
\begin{aligned}
\boldsymbol{i}_t &= \sigma(\mathbf{W}_{xi}\boldsymbol{x}_t + \mathbf{W}_{hi}\boldsymbol{h}_{t-1} + \mathbf{W}_{ci}\boldsymbol{c}_{t-1} + \boldsymbol{b}_i) \\
\boldsymbol{f}_t &= \sigma(\mathbf{W}_{xf}\boldsymbol{x}_t + \mathbf{W}_{hf}\boldsymbol{h}_{t-1} + \mathbf{W}_{cf}\boldsymbol{c}_{t-1} + \boldsymbol{b}_f) \\
\boldsymbol{c}_t &= \boldsymbol{f}_t \circ \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \circ \tanh(\mathbf{W}_{xc}\boldsymbol{x}_t + \mathbf{W}_{hc}\boldsymbol{h}_{t-1} + \boldsymbol{b}_c) \\
\boldsymbol{o}_t &= \sigma(\mathbf{W}_{xo}\boldsymbol{x}_t + \mathbf{W}_{ho}\boldsymbol{h}_{t-1} + \mathbf{W}_{co}\boldsymbol{c}_t + \boldsymbol{b}_o) \\
\boldsymbol{h}_t &= \boldsymbol{o}_t \circ \tanh(\boldsymbol{c}_t)
\end{aligned}
$$



where $\circ$ is the element-wise multiplication, $\{\boldsymbol{W}.\}$ and $\{\boldsymbol{b}.\}$ are parameters. [Graves, 2013]

Reference

Bengio, Y., Ducharme, R., and Vincent, P. (2001).
A neural probabilistic language model.
In *NIPS*.

Cho, K., Van Merriënboer, B., Bahdanau, D., and Bengio, Y. (2014).
On the properties of neural machine translation: Encoder-decoder approaches.
arXiv preprint arXiv:1409.1259.

Graves, A. (2013).
Generating sequences with recurrent neural networks.
arXiv preprint arXiv:1308.0850.

Hochreiter, S. and Schmidhuber, J. (1997).
Long short-term memory.
Neural computation, 9(8):1735–1780.

Pascanu, R., Mikolov, T., and Bengio, Y. (2013).
On the difficulty of training recurrent neural networks.
In *International Conference on Machine Learning*, pages 1310–1318.

Rumelhart, D. E., Hinton, G. E., and Williams, R. J. (1985).
Learning internal representations by error propagation.
Technical report, California Univ San Diego La Jolla Inst for Cognitive Science.

Shalev-Shwartz, S. and Ben-David, S. (2014).
Understanding machine learning: From theory to algorithms.
Cambridge university press.