CS 6501 Natural Language Processing

Statistical Language Modeling

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Overview

Introduction

Word Prediction with Context

Consider the example, what words are likely to follow

```
Please turn your homework ...
```

[Jurafsky and Martin, 2019]

Word Prediction with Context

Consider the example, what words are likely to follow

`Please turn your homework ...`

Although we do not know the actual word in the original text, we have a good sense about what of these following words are likely to follow

- `in`
- `over`
- `refrigerator`
- `the`

[Jurafsky and Martin, 2019]

Mathematical Formulation

- Let $X_1, X_2, \ldots, X_{t-1}$ be the random variables representing the words in the context, and X_t be the next word that we would like the model to predict.

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$$P(X_t \mid X_1, \ldots, X_{t-1}) \quad \text{or} \quad P(X_t \mid \boldsymbol{X}_{1:t-1}) \tag{1}$$

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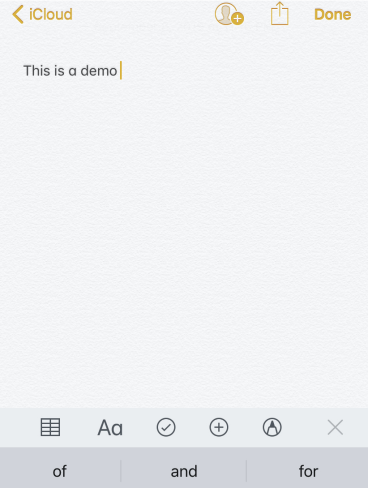
- The topics in this and the next lectures will offer two modeling methods in equation 1.
- Difference with the word embedding methods discussed in the previous lecture
  - Skip-gram model: predicting the surrounding words
  - Language models: predicting the next word

Word Prediction in Input Methods

Input methods use language models to predict the next likely words, to speed up the typing



Writing a Poem?

Trevor Noah and Amanda Gorman writing poems with the input methods on their phones

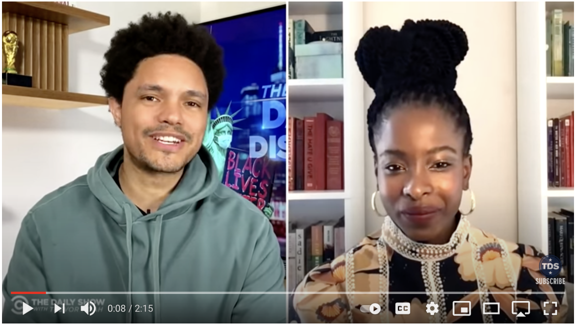


Figure: The Daily Social Distancing Show: Bonus Track feat. Amanda Gorman

Link

Joint Probability and Chain Rule

Given the conditional probability $P(X_t \mid \boldsymbol{X}_{1:t-1})$, to evaluate the qualty of a text, we need the chain rule in probability to factorize the joint probability $P(\boldsymbol{X}_{1:t})$ into a series of conditional probabilities.

$$P(X_1, X_2, \cdots, X_t) \quad = \quad P(X_1)P(X_2, \cdots, X_k \mid X_1) \tag{2}$$

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&= P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1, X_2) \cdots \\
&\quad\; P(X_t \mid X_1, \cdots, X_{t-1}) \\
&= \prod_{i=1}^{k} P(X_i \mid X_1, \ldots, X_{i-1}) \qquad (2)
\end{aligned}
$$

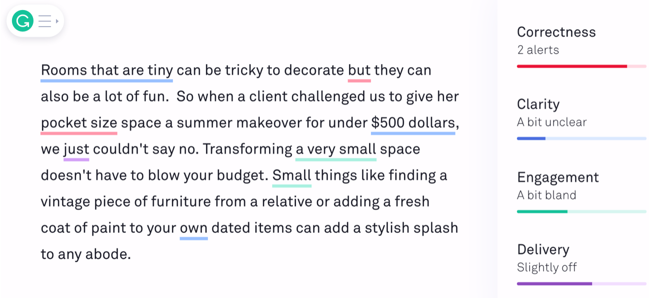
Given a voice signal, a language model in speech recognition will evaluate the likelihood of decoded texts

$$P(\text{I saw a van}) \gg P(\text{eyes awe of an}) \tag{3}$$

[Jurafsky and Martin, 2019]

Writing Assistant

Grammarly:



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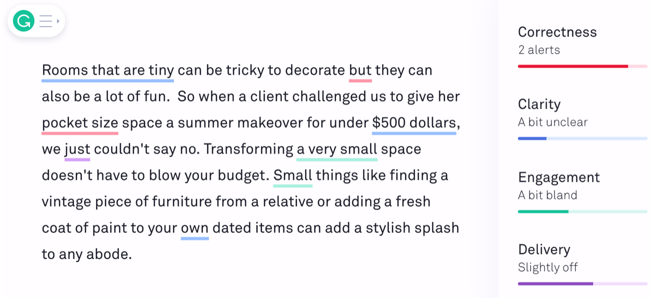

A good writing assistant system involves two tasks

- evaluate the quality of a text
- generate revision suggestions

A language model cannot provide support to all functions directly, but is a critical component in the backend system

Applications

- Generative tasks: predicting the next word given a context
  - Word prediction
  - Text generation
  - ...

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  - Word prediction
  - Text generation
  - ...
- Discriminative tasks: evaluating the quality of texts
  - Speech recognition
  - Machine translation
  - Document summarization
  - ...

N-gram Language Models

Problem Definition

Given a vocab $\mathcal{V}$ that contains all the possible word types, then the prediction of X_t can be formulated as

$$P(X_t \mid X_1, \ldots, X_{t-1}) = ? \tag{4}$$

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- it is a categorical distribution defined on the vocab $\mathscr{V}$
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- it consider the entire context from the very first word X_1 to the previous word X_{t-1}
  - The main topic of this section

Parameter Estimation

With a collection of texts as training examples, the simple method of estimating the probabilities is using maximum likelihood estimation.

- In the first lecture, we discussed the MLE of a Bernoulli distribution

$$\hat{P}(X = 1) = \frac{\sum_{i=1}^{N} \delta(x_i, 1)}{N} = \frac{c(X = 1)}{N} \tag{5}$$

  where $c(X = 1)$ is the number of observations with value 1

[Collins, 2017]

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where $c(X = 1)$ is the number of observations with value 1
- Similarly, to estimate the conditional probability $P(X_t \mid \boldsymbol{X}_{1:t-1})$, we have
$$\hat{P}(X_t = x_t \mid \boldsymbol{X}_{1:t-1} = \boldsymbol{x}_{1:t-1}) = \frac{c(\boldsymbol{x}_{1:t})}{c(\boldsymbol{x}_{1:t-1})} \quad (6)$$
where $c(\boldsymbol{x}_{1:t-1})$ is the number that text $\boldsymbol{x}_{1:t-1}$ appears in the training examples, and $c(\boldsymbol{x}_{1:t})$ is the number that text $\boldsymbol{x}_{1:t}$ appears in the training examples.

[Collins, 2017]

Parameter Estimation (Cont.)

Imagine we have a huge collection of texts for parameter estimation

- With the sentence "`the dog barks`"

$$\hat{P}(X_3 = \texttt{barks} \mid \boldsymbol{X}_{1:2} = \texttt{the dog}) = \frac{c(\boldsymbol{X}_{1:3} = \texttt{the dog barks})}{c(\boldsymbol{X}_{1:2} = \texttt{the dog})} \quad (7)$$

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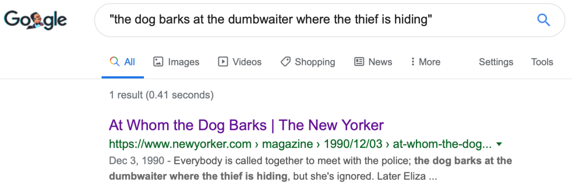
- With the sentence "`the dog barks at the dumbwaiter where the thief is hiding`"

$$\begin{aligned}\hat{P}(X_{11} &= \texttt{hiding} \mid \boldsymbol{X}_{1:10} = \texttt{the dog} \cdots \texttt{is}) \\ &= \frac{c(X_{1:11} = \texttt{the dog} \cdots \texttt{is hiding})}{c(X_{1:10} = \texttt{the dog} \cdots \texttt{is})}\end{aligned} \quad (8)$$

Parameter Estimation (Cont.)

For this specific sentence, we only one training example even if we collect all the texts from the Internet



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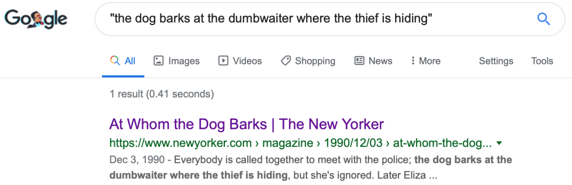

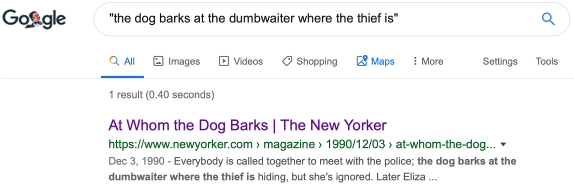

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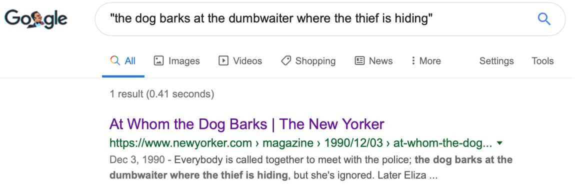

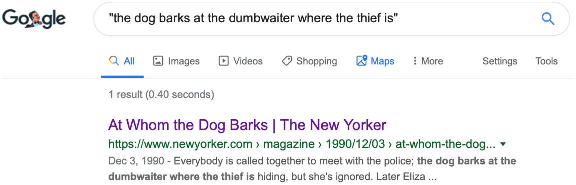

$$\hat{P}(X_{11} = \texttt{hiding} \mid \boldsymbol{X}_{1:10} = \texttt{the dog} \cdots \texttt{is}) = 1.0$$

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- Comments: the tradeoff between prediction power and number of parameters
  - It has extremely limited prediction power
  - Number of parameters: $V = |\mathcal{V}|$

Bi-gram Models

- To find a good balance between the prediction power and parameter estimation challenge, we can limit the contextual information used in a language modeling.
- Bi-gram model: uses only one word X_{t-1} from the previous context to predict the current word X_t

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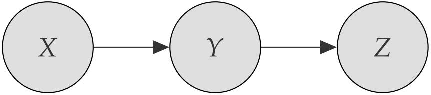
- In probabilistic modeling, a bi-gram model is an application of the first-order Markov model

Markov Property

First-order Markov property: given



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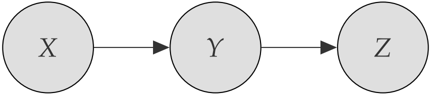

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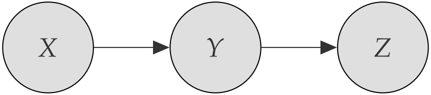

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and also the joint probability

$$\begin{aligned} P(X_1, \ldots, X_t) \quad &\approx \quad P(X_t \mid X_{t-1}) \cdot P(X_{t-1} \mid X_{t-2}) \cdots \\ & \quad\; P(X_2 \mid X_1) \cdot P(X_1) \end{aligned} \tag{15}$$

Special Tokens

Consider the application of using a bi-gram model

$$
\begin{aligned}
P(\text{the dog barks}) \quad = \quad & P(\text{the}) \cdot P(\text{dog} \mid \text{the}) \\
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Factorization with special tokens:

$$\begin{aligned} P(\square \text{ the dog barks } \blacksquare) &= P(\text{the} \mid \square) \cdot P(\text{dog} \mid \text{the}) \\ & \quad P(\text{barks} \mid \text{dog}) \cdot P(\blacksquare \mid \text{barks}) \end{aligned}$$

Example: Parameter Estimation

Example sentences

- □ I am Sam ■
- □ Sam I am ■
- □ I do not like green eggs and ham ■

[Jurafsky and Martin, 2019]

Example: Parameter Estimation

Example sentences

- `□ I am Sam ■`
- `□ Sam I am ■`
- `□ I do not like green eggs and ham ■`

Some of the probabilities:

$$\hat{P}(\mathrm{I} \mid \square) = \frac{2}{3} \quad \hat{P}(\blacksquare \mid \mathrm{Sam}) = \frac{1}{2} \quad \hat{P}(\mathrm{do} \mid \mathrm{I}) = \frac{1}{3} \tag{16}$$

[Jurafsky and Martin, 2019]

Issues with a Fixed Vocabulary

- $P(X_t \mid X_{t-1})$ is defined a fixed vocabulary, for normalization purpose

$$P(X_t \mid X_{t-1}) = \frac{c(X_{t-1}, X_t)}{\sum_{X' \in \mathcal{V}} c(X_{t-1}, X')} \tag{17}$$

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- Issues with a fixed vocabulary
  - Unknown words: word x is not in the vocabulary
  - Zero probability: word combination (x, x') never appears in the training set

Unknown Words

Replace all words that are not in the vocab with a special token UNK.

For example

- Original text: "`the dog barks at the dumbwaiter where the thief is hiding`"
- After preprocessing: "`the dog barks at the` UNK `where the thief is hiding`"

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Quiz

Can we simply ignore the unknown words? For example, what if the preprocessed text is

"`the dog barks at the  where the thief is hiding`"

Tri-gram Models

We can extend the conditional probability to depend on previous two tokens

$$P(X_t \mid X_1, \ldots, X_{t-1}) \approx P(X_t \mid X_{t-2}, X_{t-1}) \tag{18}$$

Comments

- More dependency leads to more accurate predictions
- Parameter estimation

$$\hat{P}(X_t \mid X_{t-2}, X_{t-1}) = \frac{c(\boldsymbol{X}_{t-2:t})}{c(\boldsymbol{X}_{t-2:t-1})} \tag{19}$$

Number of Parameters

- Uni-gram model
  - Ignore context words completely $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t)$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|)$

	X_1	$\cdots$	$X_{|\mathcal{V}|}$
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- Bi-gram model
  - Use only the adjacent word $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t \mid X_{t-1})$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|^2)$

$P(X_t \mid X_{t-1})$	X_1	$\cdots$	$X_{\lvert\mathcal{V}\rvert}$
X_1			
$\vdots$			
$X_{\lvert\mathcal{V}\rvert}$			

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X_1			
$\vdots$			
$X_{|\mathcal{V}|}$			

- Tri-gram model
  - Use two preceding words $P(X_t \mid \boldsymbol{X}_{1:t-1}) \approx P(X_t \mid X_{t-2}, X_{t-1})$
  - Number of parameters $\mathcal{O}(|\mathcal{V}|^3)$

Generation with Bi-gram Models

Demo Code

- A bi-gram model with no smoothing
- Training with the dataset from the arXiv paper abstracts
- Generating by *randomly* sampling from this bi-gram model

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Checkout the demo code for some examples

Smoothing Techniques

High-order Markov Models

A motivating example:

`The printer on the 5th floor of Rice hall crashed`

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n-gram Language Models

- Uni-gram: $P(X_t)$
- Bi-gram: $P(X_t \mid X_{t-1})$
- Tri-gram: $P(X_t \mid X_{t-2}, X_{t-1})$
- 4-gram: $P(X_t \mid X_{t-3}, X_{t-2}, X_{t-1})$
- 5-gram: $P(X_t \mid X_{t-4}, X_{t-3}, X_{t-2}, X_{t-1})$

Discounting

It is the same method used in parameter estimation of naive Bayes classifiers

$$P(X_t \mid X_{t-1}) = \frac{c(X_{t-1}, X_t) + \alpha}{c(X_{t-1}) + \alpha V} \tag{20}$$

where $\alpha > 0$ is a hyper-parameter.

Linear Interpolation

Estimate the following three models with MLE:

- Uni-gram: $P(X_t)$
- Bi-gram: $P(X_t \mid X_{t-1})$
- Tri-gram: $P(X_t \mid X_{t-2}, X_{t-1})$

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Then, the new probability of X_t given X_{t-2} and X_{t-1} is

$$\begin{aligned} P_{LI}(X_t \mid X_{t-2}, X_{t-1}) &= \lambda_1 \cdot P(X_t) + \lambda_2 \cdot P(X_t \mid X_{t-1}) \\ &\quad + \lambda_3 \cdot P(X_t \mid X_{t-2}, X_{t-1}) \end{aligned} \tag{21}$$

$\{\lambda_i\}$ are learned with a held-out corpus (a development set).

Language Model Evaluation

Sentence Evaluation (I)

Evaluation with joint probabilities

$$P(\text{I love black coffee}) \ \text{vs.} \ P(\text{black coffee pleases me}) \tag{22}$$

Direct comparison between the probabilities will tell us which sentence is more *fluent*.

Sentence Evaluation (II)

Limitation of comparing joint probabilities directly

$$P(\text{I love black coffee}) \text{ vs. } P(\text{I like black coffee very much}) \tag{23}$$

Due to the *length difference*, the second probability may always be smaller than the first.

Likelihood

- Test data: including the special tokens

$$\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M$$

- Likelihood

$$\begin{aligned}
\text{Log-lik}(\{\boldsymbol{x}_{m=1}^{M}\}) &= \log_2 \prod_{m=1}^{M} \prod_{t=1} P(x_{m,t} \mid \boldsymbol{x}_{m,1:t-1}) && (24) \\
&= \sum_{m=1}^{M} \sum_{t=1} \log_2 P(x_{m,t} \mid \boldsymbol{x}_{m,1:t-1}) && (25)
\end{aligned}$$

- Factors
  - Number of the tokens
  - No intuitive explanation

Perplexity

The definition of perplexity is

$$\text{Perplexity} = 2^{-\frac{1}{T}\text{Log-lik}(\{\boldsymbol{x}_{m=1}^{M}\})} \tag{26}$$

where T is the total number of the log probabilities in Log-lik($\{\boldsymbol{x}_{m=1}^{M}\}$).

Special Case

- An impossible case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = 1 \tag{27}$$

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- An impossible case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = 1 \tag{27}$$

- Perplexity

$$\begin{aligned} \text{Perplexity} &= 2^{-\frac{1}{T}\sum_{k=1}^{M}\sum_{m=1}\log_2 1} \\ &= 2^0 \\ &= 1 \end{aligned} \tag{28}$$

Special Case (II)

- A trivial case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = \frac{1}{|\mathcal{V}|} \tag{29}$$

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- A trivial case

$$P(x_t \mid \boldsymbol{x}_{1:t-1}) = \frac{1}{|\mathcal{V}|} \tag{29}$$

- Perplexity

$$\begin{aligned}
\text{Perplexity} &= 2^{-\frac{1}{T}\sum_{k=1}^{M}\sum_{m=1}\log_2 \frac{1}{|\mathcal{V}|}} \\
&= 2^{-\frac{1}{T}(T \cdot \log_2 \frac{1}{|\mathcal{V}|})} \\
&= 2^{-\log_2 \frac{1}{|\mathcal{V}|}} \\
&= |\mathcal{V}|
\end{aligned} \tag{30}$$

Typical Values of Perplexity

- $|\mathcal{V}| = 50K$
- A uni-gram model: Perplexity = 955
- A bi-gram model: Perplexity = 137
- A tri-gram model: Perplexity = 74

Lower is better

[Collins, 2017]

A Few Comments on Perplexity

Perplexity

- is an intrinsic evaluation measurement

A Few Comments on Perplexity

Perplexity

- is an intrinsic evaluation measurement
- is not necessarily correlated with the performance of
  - e.g., lower perplexity does not mean better translation (wrt BLEU score)
- is not directly comparable even on the same test data
  - you need the exactly same input for comparison

Reference

Collins, M. (2017).
Natural language processing: Lecture notes.

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